

Neutrino oscillations in de Sitter space-time

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Abstract

We try to understand flavor oscillations and to develop the formulae for describing neutrino oscillations in de Sitter space-time. First, the covariant Dirac equation is investigated under the conformally flat coordinates of de Sitter geometry. Then, we obtain the exact solutions of the Dirac equation and indicate the explicit form of the phase of wave function. Next, the concise formulae for calculating the neutrino oscillation probabilities in de Sitter space-time are given. Finally, The difference between our formulae and the standard result in Minkowski space-time is pointed out.

Key words: Neutrino oscillation, de Sitter space-time, Dirac equation

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1 Introduction

Neutrino oscillations have been the most natural explanation to the so-called solar neutrino problem and atmospheric neutrino problem [1]. Since neutrino oscillations may provide a tool both to understand elementary particles and their interactions and to uncover the secrets on ancient celestial bodies, they have attracted a lot of attention in modern physics [2]. In Minkowski space-time, neutrino oscillations have been extensively studied in the past by using both plane waves [3] and wave packets [4,5] to represent the emitted neutrinos. But, the neutrinos emitted by ancient sources must travel a very long distance to arrive at our detectors, and be affected by gravitation. Therefore, in more recent years, physicists have turned their attention to specifically gravitational contributions to neutrino oscillations.

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According to the methods appeared in the literature, we divide those considerations into three categories:

- (1) Using approximate operator [6,7,8,9]. In these papers, the effective Hamiltonian or the semiclassical approximation, such as, $p_x \simeq E - \frac{m^2}{2E}$, is generally used to investigate the complicated physical system. This kind of method is fit for considering matter effects on neutrino oscillations.
- (2) Adopting semiclassical phase factor [10,11,12,13]. This category of study first appeared in Ref.[10]. All of these papers laid their foundation on an extended covariant phase factor, for instance, $\Phi = \int_A^B p_\mu dx^\mu$, given in Ref.[14]. Since the semiclassical action is taken as the quantum phase, couplings between spin and gravitation are thus ignored.
- (3) Studying the covariant Dirac equation [15]. It seems that this is the unique way to acquire the exact formulae for calculating the oscillation probabilities. But, only in several ideal models, one can obtain the exact solutions of the Dirac equation. So this method can be used in a few cases.

Recent astronomical observations on Type Ia supernova [16,17,18] and the cosmological microwave background radiation [19,20,21] imply that dark energy can not be zero, and which will dominate the doom of our Universe. Therefore, our observable universe will be an asymptotical de Sitter space-time. On the other hand, the scenarios of inflation usually need an inflationary epoch, in which space-time can be treated as a de Sitter manifold [22]. Upon above considerations, we investigate neutrino oscillations in de Sitter space-time in this paper.

The different evolution of the phases of mass eigenstates plays the pivotal role in neutrino oscillations. In Minkowski space-time, the phase of a free boson or of a free fermion is given by solving the Klein-Gordon equation or Dirac equation, that is, $\Phi(t, \vec{x}) = -(E \cdot t - \vec{p} \cdot \vec{x})$. From this point of view, we need to solve the covariant Dirac equation to get the exact form of the phase of wave function. After that, we can obtain the exact formulae for calculating the neutrino oscillation probabilities.

This paper is organized as follows. The covariant Dirac equation in de Sitter manifold is explicitly given in section 2. In section 3, the exact solutions of the Dirac equation are obtained. After that, we read out the phase factor. We then present the formulae on neutrino-oscillation probabilities in section 4. Conclusions and remarks appear in the last section.

2 Dirac equation in de Sitter space-time

To set the stage for studying neutrino oscillations in a curved space-time, we introduce the local tetrad field as follows¹

$$g^{\mu\nu} = e^\mu_a(x) e^\nu_b(x) \eta^{ab} , \quad (1)$$

where $g^{\mu\nu}$ being the space-time metric, and the metric tensor of Minkowski space-time η_{ab} satisfy

$$\eta^{00} = +1 , \quad \eta^{11} = \eta^{22} = \eta^{33} = -1 , \quad \eta^{ab} = 0 \quad \text{for} \quad a \neq b . \quad (2)$$

In a curved space-time, the covariant Dirac equation reads²

$$i\gamma^\mu(x) \left[\frac{\partial}{\partial x^\mu} - \Gamma_\mu(x) \right] \psi(x) = m\psi(x) . \quad (3)$$

Here $\gamma^\mu(x)$ are the curvature-dependent Dirac matrices and $\Gamma_\mu(x)$ are the spin connections to be determined. The curvature-dependent Dirac matrices $\gamma^\mu(x)$ are presented in terms of the tetrad field as

$$\gamma^\mu(x) = e^\mu_a(x) \gamma^a , \quad (4)$$

where γ^a denotes the standard flat-space Dirac matrices, which satisfies $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$.

Under a general coordinate transformation, $x^\mu \longrightarrow y^\nu$, it is well known that the spinor keeps invariant [23], that is,

$$\psi(y) = \psi(x) . \quad (5)$$

Hence the phase difference of two points for a free neutral spin- $\frac{1}{2}$ particle is invariant under the coordinate transformation. As we will demonstrate, the phenomenon of flavor oscillation is only related with the phase difference and the mixing angles, one then can select any convenient coordinates to study flavor oscillation. Because of this property, we choose the comoving coordinates to investigate the neutrino oscillations in de Sitter space-time.

¹ We use Roman suffixes to refer to the bases of local Minkowski frame, and use Greek suffixes to refer to curvilinear coordinates of space-time.

² we adopt the conventions that $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ and $\hbar = c = 1$.

In the conformally flat coordinates the metric of de Sitter space-time is taken to be

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - R^2(t) \left[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] , \quad (6)$$

where $R(t) = e^{Ht}$ is called cosmic scale factor of de Sitter space-time. H is a constant.

The tetrad field in the case of de Sitter metric (6) is thus taken to be

$$e^\mu_a(x) = \text{diag} \left(1, \frac{1}{R}, \frac{1}{Rr}, \frac{1}{Rr \sin \theta} \right) . \quad (7)$$

Inserting Eq.(7) into Eq.(4) yields

$$\begin{aligned} \gamma^0(x) &= \gamma_0 , & \gamma^1(x) &= -\frac{1}{R} \gamma_1 , \\ \gamma^2(x) &= -\frac{1}{Rr} \gamma_2 , & \gamma^3(x) &= -\frac{1}{Rr \sin \theta} \gamma_3 . \end{aligned} \quad (8)$$

The spin connections $\Gamma_\mu(x)$ in Eq.(3) satisfy the equation

$$[\Gamma_\mu(x), \gamma^\nu(x)] = \frac{\partial \gamma^\nu(x)}{\partial x^\mu} + \Gamma_{\mu\rho}^\nu \gamma^\rho(x) , \quad (9)$$

where $\Gamma_{\mu\rho}^\nu$ are the Christoffel symbols for the metric (6), which are determined by

$$\Gamma_{\mu\rho}^\nu = \frac{1}{2} g^{\nu\tau} \left(\frac{\partial g_{\tau\rho}}{\partial x^\mu} + \frac{\partial g_{\mu\tau}}{\partial x^\rho} - \frac{\partial g_{\mu\rho}}{\partial x^\tau} \right) . \quad (10)$$

Hence Eq.(9) can be solved for the spin connections $\Gamma_\mu(x)$ which we determine as

$$\begin{aligned} \Gamma_0 &= 0 , & \Gamma_1 &= \frac{\dot{R}}{2} \gamma_0 \gamma_1 , \\ \Gamma_2 &= \frac{1}{2} \dot{R} r \gamma_0 \gamma_2 + \frac{1}{2} \gamma_2 \gamma_1 , \\ \Gamma_3 &= \frac{1}{2} \dot{R} r \sin \theta \gamma_0 \gamma_3 + \frac{1}{2} \sin \theta \gamma_3 \gamma_1 + \frac{1}{2} \cos \theta \gamma_3 \gamma_2 , \end{aligned} \quad (11)$$

where \dot{R} denotes $\frac{dR(t)}{dt}$, so that the combination $\gamma^\mu(x)\Gamma_\mu(x)$ in Eq.(3) simplifies to

$$\gamma^\mu(x)\Gamma_\mu(x) = -\frac{3\dot{R}}{2R}\gamma_0 + \frac{1}{Rr}\gamma_1 + \frac{\cos\theta}{2Rr\sin\theta}\gamma_2 . \quad (12)$$

Inserting Eq.(8) and Eq.(12) into Eq.(3), then we obtain the Dirac equation in de Sitter space-time

$$\left\{ iR\gamma_0 \left(\frac{\partial}{\partial t} + \frac{3\dot{R}}{2R} \right) - i \left[\gamma_1 \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) + \frac{1}{r}\gamma_2 \left(\frac{\partial}{\partial \theta} + \frac{\cos\theta}{2\sin\theta} \right) + \frac{1}{r\sin\theta}\gamma_3 \frac{\partial}{\partial \varphi} \right] - mR \right\} \psi(t, r, \theta, \varphi) = 0 . \quad (13)$$

3 The exact solutions of the Dirac equation

We can simplify the Dirac equation (13) by transforming [24,25]

$$\psi(t, r, \theta, \varphi) = (\sin\theta)^{-\frac{1}{2}} R^{-\frac{3}{2}} \Psi(t, r, \theta, \varphi) . \quad (14)$$

The reduced equation in terms of $\Psi(t, r, \theta, \varphi)$ is of the form

$$R \left(\frac{\partial}{\partial t} + im\gamma_0 \right) \Psi = \left[\gamma_0\gamma_1 \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) + \frac{1}{r}\gamma_1\hat{K}(\theta, \varphi) \right] \Psi , \quad (15)$$

where

$$\hat{K}(\theta, \varphi) = \gamma_0\gamma_1 \left(\gamma_2 \frac{\partial}{\partial \theta} + \gamma_3 \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \right) \quad (16)$$

is a Hermitian operator, as we know, which is defined in Ref.[26] first. The eigen equation of \hat{K} is

$$\hat{K}\Psi_{\varsigma,\kappa} = \varsigma\Psi_{\varsigma,\kappa} , \quad \varsigma = 0, \pm 1, \pm 2, \dots , \quad (17)$$

where the eigen value ς is an integer, which has been proven in Ref.[23,26].

Only the two matrices γ_0 and γ_1 remain explicitly in the simplified Dirac equation (15), they can therefore be represented by 2×2 matrices

$$\gamma_0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & -\tau_1 \\ \tau_1 & 0 \end{pmatrix}, \quad (18)$$

where $\mathbf{1}$, τ_1 are respectively 2×2 unit matrix and the Pauli matrix, namely

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (19)$$

We separate the angular factor from the wave function, and represent the radial and temporal factor by a two-component spinor, that is

$$\Psi_{\varsigma, \kappa} = \Theta_{\varsigma}(\theta, \varphi) \begin{pmatrix} \phi(r, t) \\ \chi(r, t) \end{pmatrix}, \quad (20)$$

where the angular factor, $\Theta_{\varsigma}(\theta, \varphi)$, is determined by the requirement $\hat{K}\Theta_{\varsigma}(\theta, \varphi) = \varsigma\Theta_{\varsigma}(\theta, \varphi)$. This equation for eigenstates of the angular motion has been investigated by Schrödinger [26]. He found, the operator \hat{K} is related to the total angular momentum. Their eigenvalues satisfy [27]

$$\varsigma = \mp(j + \frac{1}{2}) = \begin{cases} -(l+1) & \text{for } j = l + \frac{1}{2} \\ l & \text{for } j = l - \frac{1}{2} \end{cases}. \quad (21)$$

Also the eigenfunction Θ_{ς} is related with the spherical harmonics as follows

$$\Theta_{\varsigma}(\theta, \varphi) \propto Y_{l, \overline{m} + \frac{1}{2}}(\theta, \varphi) \propto P_l^{\overline{m} + \frac{1}{2}}(\cos \theta) e^{i(\overline{m} + \frac{1}{2})\varphi}, \quad (22)$$

or

$$\Theta_{\varsigma}(\theta, \varphi) \propto Y_{l, \overline{m} - \frac{1}{2}}(\theta, \varphi) \propto P_l^{\overline{m} - \frac{1}{2}}(\cos \theta) e^{i(\overline{m} - \frac{1}{2})\varphi}, \quad (23)$$

where $P(\cos \theta)$ being the associated Legendre functions, and $\overline{m} \pm \frac{1}{2} = 0, \pm 1, \pm 2, \dots, \pm l$.

With the help of equations (17), (18), and (20), we can rewrite the equation (15) into two equations

$$R \left(\frac{\partial}{\partial t} + im \right) \phi(r, t) = \left[\left(\frac{\partial}{\partial r} + \frac{1}{r} \right) + \frac{\varsigma}{r} \right] \tau_1 \chi(r, t) , \quad (24)$$

$$R \left(\frac{\partial}{\partial t} - im \right) \chi(r, t) = \left[\left(\frac{\partial}{\partial r} + \frac{1}{r} \right) - \frac{\varsigma}{r} \right] \tau_1 \phi(r, t) . \quad (25)$$

Here $\frac{1}{r}\tau_1\varsigma$ is the term of spin-orbit coupling. To solve the above equations, we separate the functions $\phi(t, r)$ and $\chi(t, r)$ into radial and temporal factors, respectively

$$\phi(t, r) = U_1(r)T_1(t) , \quad \chi(t, r) = U_2(r)T_2(t) . \quad (26)$$

With the help of Eq.(19), inserting the above equations into Eq.(24) and Eq.(25) yields the evolution equations

$$R \left(\frac{d}{dt} + im \right) T_1(t) - i\kappa T_2(t) = 0 , \quad (27)$$

$$R \left(\frac{d}{dt} - im \right) T_2(t) - i\kappa T_1(t) = 0 , \quad (28)$$

and the radial equations

$$\left[\left(\frac{d}{dr} + \frac{1}{r} \right) + \frac{\varsigma}{r} \right] \left[\left(\frac{d}{dr} + \frac{1}{r} \right) - \frac{\varsigma}{r} \right] U_1 + \kappa^2 U_1 = 0 , \quad (29)$$

$$\left[\left(\frac{d}{dr} + \frac{1}{r} \right) - \frac{\varsigma}{r} \right] \left[\left(\frac{d}{dr} + \frac{1}{r} \right) + \frac{\varsigma}{r} \right] U_2 + \kappa^2 U_2 = 0 . \quad (30)$$

Obviously, if U_1 is substituted by U_2 , and simultaneously ς are replaced by $-\varsigma$, then the equation (29) becomes the equation (30). Therefore, in the following, it is of only necessity to study the equation (29).

The equation (29) directly simplifies to [25]

$$\frac{d^2 U_1}{dr^2} + \frac{2}{r} \frac{dU_1}{dr} + \left(\frac{\varsigma(1 - \varsigma)}{r^2} + \kappa^2 \right) U = 0 . \quad (31)$$

Through a simple transformation, one can easily find that the equation (31) is the Bessel's equation of order $-\varsigma + \frac{1}{2}$. The solutions of the above equation

are [28]

$$U_1(r) = \frac{1}{\sqrt{\kappa r}} J_{\pm(\varsigma - \frac{1}{2})}(\kappa r) . \quad (32)$$

Where $J_{\pm(\varsigma - \frac{1}{2})}(\kappa r)$ are called the Bessel functions. In the case of $\varsigma = 0, -1, -2, \dots$, the Bessel functions in terms of sine or cosine functions are explicitly taken to be [28]

$$\begin{aligned} J_{-\varsigma + \frac{1}{2}}(\kappa r) = & \sqrt{\frac{2}{\pi \kappa r}} \left\{ \sin\left(\kappa r + \frac{\varsigma \pi}{2}\right) \sum_{n=0}^{[\frac{-\varsigma}{2}]} \frac{(-1)^n (-\varsigma + 2n)!}{(2n)!(-\varsigma - 2n)!(2\kappa r)^{2n}} \right. \\ & \left. + \cos\left(\kappa r + \frac{\varsigma \pi}{2}\right) \sum_{n=0}^{[\frac{-\varsigma+1}{2}]} \frac{(-1)^n (-\varsigma + 2n + 1)!}{(2n+1)!(-\varsigma - 2n - 1)!(2\kappa r)^{2n+1}} \right\} , \end{aligned} \quad (33)$$

and

$$\begin{aligned} J_{\varsigma - \frac{1}{2}}(\kappa r) = & \sqrt{\frac{2}{\pi \kappa r}} \left\{ \cos\left(\kappa r - \frac{\varsigma \pi}{2}\right) \sum_{n=0}^{[\frac{-\varsigma}{2}]} \frac{(-1)^n (-\varsigma + 2n)!}{(2n)!(-\varsigma - 2n)!(2\kappa r)^{2n}} \right. \\ & \left. - \sin\left(\kappa r - \frac{\varsigma \pi}{2}\right) \sum_{n=0}^{[\frac{-\varsigma+1}{2}]} \frac{(-1)^n (-\varsigma + 2n + 1)!}{(2n+1)!(-\varsigma - 2n - 1)!(2\kappa r)^{2n+1}} \right\} . \end{aligned} \quad (34)$$

The formulae for $\varsigma > 0$ are easy to read out from above two equations.

We have indicated that the solution of U_2 can be obtained from the solution (32) by substituting $-\varsigma$ for ς . Hence we have acquired the exact forms of the spatial factor of wave function in de Sitter geometry.

4 The factor of time evolution

After setting $s = 1/R(t)$, since $R(t) = e^{Ht}$, it is easy to get the following formula

$$H = \frac{1}{R} \frac{dR}{dt} = -\frac{1}{s} \frac{ds}{dt} . \quad (35)$$

In terms of new variable s , the evolution equations (27) and (28) can be rewritten as

$$-HT_1' + \frac{im}{s}T_1 - i\kappa T_2 = 0 , \quad (36)$$

$$-HT_2' - \frac{im}{s}T_2 - i\kappa T_1 = 0 , \quad (37)$$

where $T' \equiv dT/ds$. From the above equations, we acquire the equation satisfied by T_1 as follows

$$s^2 T_1'' + \left[\frac{\kappa^2 s^2}{H^2} - \nu(\nu - 1) \right] T_1 = 0 , \quad (38)$$

where

$$\nu = \frac{im}{H} \quad \text{or} \quad \nu = 1 - \frac{im}{H} . \quad (39)$$

Obviously, the above equation is the Bessel's equation of order $\nu - \frac{1}{2}$, the solutions of which is generally presented as

$$T_1(t) = \frac{1}{\sqrt{R}} J_{\pm(\nu-\frac{1}{2})} \left(\frac{\kappa}{HR} \right) . \quad (40)$$

These solutions demonstrate that $\nu = \frac{im}{H}$ and $\nu = 1 - \frac{im}{H}$ are quite equivalent. If 2ν isn't an integer, then the Bessel function $J_{\pm(\nu-\frac{1}{2})}$ can be explicitly expressed as the infinite series, that is

$$\begin{aligned} J_{\nu-\frac{1}{2}} \left(\frac{\kappa}{HR} \right) &= \sum_{\iota=0}^{\infty} \frac{(-1)^{\iota}}{\iota!} \frac{1}{\Gamma(\nu + \iota + \frac{1}{2})} \left(\frac{\kappa}{2HR} \right)^{2\iota + \nu - \frac{1}{2}} \\ &= \left(\frac{\kappa}{2H} \right)^{i\frac{m}{H}} e^{-imt} \sum_{\iota=0}^{\infty} \frac{(-1)^{\iota}}{\iota!} \frac{1}{\Gamma(\nu + \iota + \frac{1}{2})} \left(\frac{\kappa}{2HR} \right)^{2\iota - \frac{1}{2}} , \end{aligned} \quad (41)$$

where $\Gamma(\cdots)$ is the gamma function. It is easy to read out the form of $T_2(t)$

$$T_2(t) = \frac{1}{\sqrt{R}} J_{\pm(-i\frac{m}{H}-\frac{1}{2})} \left(\frac{\kappa}{HR} \right) . \quad (42)$$

Hence we have given the exact solutions of the Dirac equation in de Sitter space-time.

In a different coordinates of spatially flat Robertson-Walker space-time, the exact solutions of the Dirac equation with $R(t) = e^{Ht}$ have been given in Ref.[29]. We acquire the same evolution factor as the result in Ref.[29].

5 Neutrino oscillations

Recent neutrino experiments imply that the masses of neutrinos satisfy $m \sim 10^{-2}\text{eV}$ [2]. If one select H as the present-day Hubble constant, then $|\nu|$ is about 10^{30} . Therefore, $|\nu|$ is a huge number. Then we can acquire the approximate formulae of the Bessel function from Eq.(41)

$$J_{\nu-\frac{1}{2}}\left(\frac{\kappa}{HR}\right) \sim \exp\left\{\nu - \frac{1}{2} + \left(\nu - \frac{1}{2}\right) \log\left(\frac{\kappa}{2HR}\right) - \nu \log\left(\nu - \frac{1}{2}\right)\right\} \\ \cdot \left[\frac{1}{\sqrt{2\pi}} + \frac{c_1}{\nu - \frac{1}{2}} + \frac{c_2}{(\nu - \frac{1}{2})^2} + \cdots\right], \quad (43)$$

where c_1 and c_2 are small constants. From above equation, we can reasonably obtain more concise expression as follows

$$J_{\nu-\frac{1}{2}}\left(\frac{\kappa}{HR}\right) \sim \frac{1}{\sqrt{2\pi}} \exp\left\{\nu + \nu \left(\log\left(\frac{\kappa}{2im}\right) - Ht\right)\right\} \\ = \frac{1}{\sqrt{2\pi}} e^{-\frac{3\pi m}{2H}} \exp\left\{\frac{im}{H} \left[1 + \log\left(\frac{\kappa}{2m}\right)\right] - imt\right\}. \quad (44)$$

It is well known that the Dirac equation describes both particle and its antiparticle. So the sine or cosine functions shall be rewritten as the exponential function with imaginary argument. According to the exact solutions of the Dirac equation, we can read out the general form of the phase in de Sitter space-time, that is

$$\text{phase} = \frac{m}{H} \left[1 + \log\left(\frac{\kappa}{2m}\right)\right] - mt + \kappa r + (\overline{m} \pm \frac{1}{2})\varphi \pm \frac{\varsigma\pi}{2} + z_{\varsigma, \overline{m}}(\theta), \quad (45)$$

where $z_{\varsigma, \overline{m}}(\theta)$ is a function related with eigenvalues ς , \overline{m} and coordinate θ .

Updated large experiments have shown that neutrinos are possibly massive although their masses are very tiny. If there are flavor mixing in neutrino sector, and the leptonic quantum numbers are only approximately conserved, then there exist the fascinating possibility of neutrino oscillations.

In the case of three generations of neutrinos, each of the three neutrino mass eigenstates shall be represented by ν_I ; $I = 1, 2, 3$. Because of neutrino mixing, we have the linear superposition

$$\nu_\Lambda = \sum_{I=1,2,3} U_{\Lambda I} \nu_I, \quad (46)$$

where $U_{\Lambda I}$ is a unitary matrix, and $\Lambda = e, \mu, \tau$ represents the weak flavor eigenstates (corresponding to electron, muon and tau neutrinos respectively). The ν_1, ν_2 and ν_3 correspond to the three mass eigenstates of masses m_1, m_2 and m_3 , respectively. Conventionally the mixing matrix $U_{\Lambda I}$ is parameterized by three mixing angles, one CP violating phase and two Majorana phases. For the sake of investigation on neutrino oscillations, we ignore the CP violating phase. In this paper we assume that the neutrinos are of the Dirac type. Hence, in terms of mixing angles α, β, η , the mixing matrix reads [2]

$$U(\alpha, \beta, \eta) = \begin{pmatrix} c_\alpha c_\eta & s_\alpha c_\eta & s_\eta \\ -c_\alpha s_\beta s_\eta - s_\alpha c_\beta & c_\alpha c_\beta - s_\alpha s_\beta s_\eta & s_\beta c_\eta \\ -c_\alpha c_\beta s_\eta + s_\alpha s_\beta & -c_\alpha s_\beta - s_\alpha c_\beta s_\eta & c_\beta c_\eta \end{pmatrix}, \quad (47)$$

where $c_\xi \equiv \cos(\xi)$, $s_\xi \equiv \sin(\xi)$, with $\xi = \alpha, \beta, \eta$.

Let us assume: a weak flavor eigenstate, e.g., an electron neutrino, is emitted at a point P with coordinates $P(t_e, r_e, \theta_e, \varphi_e)$. For simplicity, we are interested in its radial propagation. We also assume that the different mass eigenstates have the same total angular momentum ς . So, in the momentum representation, the mass eigenstates must have different eigenvalue κ , denoted by κ_I . Therefore, in de Sitter space-time, the radial evolution of them is given by the expression:

$$\nu_I(t > t_e, r) = \exp \left\{ i \frac{m_I}{H} \left[1 + \log \left(\frac{\kappa_I}{2m_I} \right) \right] - im_I(t - t_e) + i\kappa_I(r - r_e) + i\Pi \right\} \nu_I(P), \quad (48)$$

here Π being the phase factor which is the same for all mass eigenstates.

Considering Eq.(47) and Eq.(48) together, one can easily draw the following conclusion: in de Sitter space-time, under the assumption that the mass eigenstates have the same total angular momentum, if a weak flavor eigenstate ν_Λ is created at the point P , then the probability to find the weak flavor eigenstate $\nu_{\Lambda'}$ at the point $Q(t_f, r_f, \theta_e, \varphi_e)$ is given by the formulae:

$$\begin{aligned} p(\nu_\Lambda \rightarrow \nu_{\Lambda'}) &= |\nu_{\Lambda'}(Q)|^2 \\ &= \sum_{I,J=1}^3 \{ U_{\Lambda' I} \exp[-im_I(t_f - t_e) + i\kappa_I(r_f - r_e)] U_{\Lambda I} \} \\ &\quad \cdot \{ U_{\Lambda' J} \exp[+im_J(t_f - t_e) - i\kappa_J(r_f - r_e)] U_{\Lambda J} \} \\ &= \delta_{\Lambda' \Lambda} - 4U_{\Lambda' 1}U_{\Lambda 1}U_{\Lambda' 2}U_{\Lambda 2} \sin^2[\omega_{12}] - 4U_{\Lambda' 1}U_{\Lambda 1}U_{\Lambda' 3}U_{\Lambda 3} \sin^2[\omega_{13}] \end{aligned}$$

$$- 4U_{\Lambda'2}U_{\Lambda2}U_{\Lambda'3}U_{\Lambda3}\sin^2[\omega_{23}] , \quad (49)$$

where

$$\omega_{IJ} = \frac{\Delta\kappa}{2}\Delta r - \frac{\Delta m}{2}\Delta t + \frac{\Delta m}{2H} + \frac{1}{2H} \log \left[\left(\frac{\kappa_J}{2m_J} \right)^{m_J} \left(\frac{2m_I}{\kappa_I} \right)^{m_I} \right] , \quad (50)$$

$$\Delta r = r_f - r_e , \quad \Delta t = t_f - t_e , \quad (51)$$

$$\Delta\kappa = \kappa_J - \kappa_I , \quad \Delta m = m_J - m_I . \quad (52)$$

We have given the formulae for calculating the neutrino oscillation probabilities in de Sitter space-time, which demonstrate that the probabilities are related with not only the positions, masses and momenta, but also the Hubble constant H . In Minkowski space-time, the Hubble constant doesn't appear.

6 Conclusion

The neutrino oscillations in de Sitter space-time is studied through investigating the covariant Dirac equation. The exact solutions of the Dirac equation are obtained in the conformally flat coordinates. The phase of the spin- $\frac{1}{2}$ wave function is read out. Then the formulae of neutrino oscillation probabilities in de Sitter space-time are constructed. Our result is quite different from the stand form in Minkowski space-time because of the different topology of de Sitter manifold.

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